

A toy model based analysis on the effect of the Lee-Wick partners in the evolution of the early universe

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Abstract

In the present article the thermodynamic results of the Lee-Wick partner infested universe have been applied in a toy model where there is one Lee-Wick partner to each of the standard model particle and more over the longitudinal degrees of freedom of the massive partners of the standard massless gauge bosons are neglected at high temperatures. For practical purposes, the chiral fermionic sector of Lee-wick theories requires two Lee-Wick partners per fermion which opens up the possibility for a negative energy density of the early universe. A toy Lee-Wick model with one fermionic partner relaxes such oddities and hence easy to deal with. In a similar way, the longitudinal degrees of freedom of the massive gauge boson partners also have the potential to yield negative energy densities and thus those will be neglected in a toy model study. In such a toy model one can analytically calculate the time-temperature relation in the very early radiation dominated universe which shows interesting new physics. The article also tries to point out how a Lee-Wick particle dominated early cosmology transforms into the standard cosmological model. Based on the results of this toy model analysis a brief discussion on the more realistic model, which can accommodate two Lee-Wick partners for each standard fermionic field and the longitudinal degree of freedom of

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partners of the gauge fields, is presented. It has been shown that such an universe is mostly very difficult to attain but there are certain conditions where one can indeed think of such an universe which can evolve into the standard cosmological universe in a short time duration.

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1 Introduction

In 1969 Lee and Wick made an attempt [1, 2] to construct a Lorentz invariant, gauge invariant, unitary and divergence-free Quantum Electrodynamics (QED) by introducing unusual partners of the normal fields in the Lagrangian. The partner fields, called conventionally as the Lee-Wick partners, lived in an indefinite metric Hilbert space. In recent years the Lee-Wick theory, mainly constructed for taming divergences in QED, was generalized to construct a Lee-Wick theory of the standard model of particle physics [3]. In the Lee-Wick standard model the authors constructed a higher-derivative version of quantum field theory and showed that the higher derivatives in the theory can be removed by introducing auxiliary Lee-Wick partner fields. It is also shown in [3] that such a quantum field theory can be taken as an extension of the standard model where the mass of the Higgs field is stable against the quadratically divergent radiative corrections and thus solves the ‘Hierarchy puzzle’. Later on various aspects of this remarkable result have been explored in several works. Some interesting applications of the Lee-Wick idea show the scope and depth of this idea. In [4] a minimal extension of Lee-Wick Standard model (LWSM) is considered analyzing its signatures in LHC, in [5] a Lee-Wick standard model is analyzed where each standard model particle is accompanied by two Lee-Wick partners, in [6] gauge-coupling unification is achieved within the framework of Lee-Wick standard model and in [7, 8] analysis of two-Higgs doublet models where one of the doublet contains Lee-Wick fields is done. In [9] the process $gg \rightarrow h_0 \rightarrow \gamma\gamma$ is studied in the framework of LWSM where it has been pointed out that small changes in the rate of these processes due to presences of Lee-Wick fields can be treated as a distinguishing feature of LWSM from other models such as universal extra dimensions. In [10] Higgs pair production processes $gg \rightarrow h_0 h_0$ and $gg \rightarrow h_0 \tilde{p}_0$ are studied in LWSM framework to point out that the LW Higgs can be seen in the LHC upgrade in 2012 particularly when the new LW Higgs states are below the top pair threshold. The Lee-Wick field theories described in [3] has also been extended to applications in cosmology where in [11] presence of Lee-Wick scalar fields in early universe leading to non-singular bouncing universe is analyzed and in [12, 13] the stability condition on presence of radiation fields and its Lee-Wick partners during such a bouncing universe is studied. Scalar perturbations in

the Lee-Wick bouncing universe [11] have been studied choosing spatially flat gauge in [14] and the corresponding power spectrum has also been analyzed.

In this work we will mainly focus on the main results of [15] and [16] where the authors have independently derived the thermodynamic properties of the Lee-Wick resonances at high-temperatures i.e. when the temperature of the fluid containing both the standard model particles and their Lee-Wick partners is much higher than the mass of the heavy Lee-Wick resonances. It is shown that the Lee-Wick fields at high temperature contribute negatively to the energy density and pressure, although the net pressure and energy density which gets contributions from the standard model particles and their Lee-Wick partners remain positive. Two different approaches were taken in these two works, [15] and [16], to derive the thermodynamic properties of Lee-Wick resonances at high temperature. As a functional integral formulation of Lee-Wick theories is not properly derived, the authors of [15] have followed a method of statistical field theory developed in [17] to calculate the thermodynamic properties of these unusual Lee-Wick partners which are treated as unstable resonances. On the other hand the authors of [16] work with a variant of Lee-Wick's original idea, first predicted by Boulware and Gross in [18]. In this formulation the unusual Lee-Wick fields, unlike Lee-Wick's initial work, live in a definite metric space but carry negative energy. Using these negative energy fields one can reproduce the same thermodynamic properties as in [15] from the first principles of statistical mechanics. Without dealing much with the intricacies of these two methods, we will follow the main results of these two works [15, 16]. It is to be noted that although, by assumption, an indefinite metric state cannot be an initial or final state of a scattering process still they may contribute to the thermodynamics of the system. In this regard one can envisage the Lee-Wick particles as highly unstable resonances arising as intermediaries in the scattering or decay processes of standard model particles as elaborated in [15]. The resonances live momentarily and can contribute to the thermodynamics of the system.

Within the standard framework of cosmological evolution, the universe evolves through different phases such as radiation dominated era (when the cosmic fluid has a state parameter $\omega = 1/3$), matter dominated era (with $\omega = 0$), or inflationary and present dark energy dominated phases of exponential expansion (with $\omega < -1/3$). On the other hand a cosmic fluid infested with Lee-Wick resonances at very high temperatures leads to an unusual equation of state ($\omega = 1$) [15, 16] which might have affected the evolution of the early universe in an unconventional way¹. This present article deals with features of such an unconventional cosmological scenario. According to Lee-Wick theories the Lee-Wick partners of the

¹Although Lee-Wick thermodynamics predict an equation of state where $\omega \sim 1$ it has been verified that for the toy model with single LW partner for each particle the speed of sound $c_s = \sqrt{dp/d\rho}$ remains smaller than one and causality is maintained [15].

standard model particles are very short-lived and can only exist as resonances. The unusual equation of state $\omega = 1$ arises when the temperature (T) of the cosmic fluid is high enough to thermalize the short-lived Lee-Wick resonances (i.e. $T \gg M$, where M is the mass of the Lee-Wick resonance). Thus we will deal with an unconventional radiation dominated era when the temperature of the universe is high enough so that all the particles, including the Lee-Wick resonances, are thermalized but the equation of state is not the standard one corresponding to the radiation dominated phase.

It is to note at this point that the Lee-Wick particles cannot occur as initial or final states in any scattering or decay process but can exist momentarily as intermediate states. They are introduced in a theory to overcome the ultraviolet divergences plaguing it. When the temperature of the system is much higher than the masses of the Lee-Wick resonances ($T \gg M$) then the normal particles can have energy-momentum greater than M and consequently the unstable resonances can actually “live” momentarily and get equilibrated. When the temperature of the universe decreases and ($T < M$) then the energy-momentum of the initial particles undergoing scattering will be less than M and eventually the Lee-Wick resonances cannot be produced momentarily, they will only act as virtual particles looping in the propagators. Thus as the temperature of the universe becomes less than the mass of a Lee-Wick partner the ephemeral Lee-Wick resonance will not get a chance to be thermally equilibrated and practically the Lee-Wick partner will thermally decouple from the plasma. A Lee-Wick partner decouples from the equilibrated thermal plasma when it loses its property to be produced momentarily as an intermediate state in a standard scattering or decay process and this happens when $T < M$. When the Lee-Wick partners decouple they simply act as virtual particles and do not contribute to energy density and pressure of the universe. In conventional cosmology we refer to decoupling of a particle from the cosmic plasma when their interaction rate (Γ) becomes less than the expansion rate (H) of the universe i.e. $\Gamma < H$. But as the Lee-Wick partners can exist only as intermediate states, they do not have such conventional interactions with the cosmic plasma. Thus the only way they can decouple from the cosmic soup when $T < M$ is by being ‘non-thermal’ as discussed above. We will show later that this way of becoming ‘non-thermal’ produces rapid temperature and entropy fluctuations of the remaining cosmic fluid which will lead the system to go out of equilibrium momentarily.

The rest of the article is presented in the following way. The next section deals with the basic thermodynamic parameters of a system where each standard particle has one Lee-Wick partner and where one neglects the longitudinal degree of freedom of the massive partners of the massless standard gauge bosons. This is more like a toy model as because in a realistic Lee-Wick standard model there are two fermionic partners for each standard chiral fermion [19, 20] and the longitudinal mode of the massive gauge boson partners cannot be neglected. We still present two sections devoted to this toy model as most of the interesting

physics which will follow in the realistic case can be qualitatively understood by the toy model thermodynamics. Section 3 deals with the cosmological effects arising out of the thermodynamics of the Lee-Wick partner infested phase. The properties of the universe where each standard boson has one Lee-Wick partner, each standard fermion has two Lee-Wick partners and the partners of the gauge bosons have three degrees of freedom are discussed in section 4. The final section presents a discussion about the major observations in the article and it ends with a brief conclusion.

2 Thermodynamic properties of the Lee-Wick partner infested universe

Lee-Wick partners of the standard model particles do not appear as initial or final states of any real scattering or decay process. This is one of the assumptions of Lee and Wick's proposal [1, 2]. The Lee-Wick partners can only appear as virtual particles or resonances. In [15] the authors claimed that the Lee-Wick resonances, which are unstable, can thermalize if the temperature of the system is much greater than their masses. The masses of the Lee-Wick partners are assumed to be much greater than the masses of the standard model particles.

In the radiation dominated phase of standard cosmology whenever the temperature of the universe scales down to the mass of a particular particle species then in general the particle species annihilates by interacting with its anti-particles. This process can happen in thermal equilibrium. The analogous picture in the Lee-Wick paradigm corresponds to the situation where the Lee-Wick resonances decouple. Later it will be shown that unlike standard particles the Lee-Wick thermal resonances decouple out of thermal equilibrium. One can say at these temperatures the Lee-Wick thermal resonances decouple from the rest of the plasma which is in thermal equilibrium. Decoupling starts when the temperature of the universe becomes equal to the mass of a Lee-Wick resonance. As the masses of the Lee-Wick partners are supposed to be much higher than their standard partners the thermal resonances of the Lee-Wick partners will start to decouple much earlier than their standard model counterparts. These events can modify the thermodynamics of the very early universe. These modifications of the thermodynamics of the early universe can have interesting cosmological impacts.

2.1 The energy density and entropy density of the Lee-Wick particle infested universe

In the standard model of cosmological evolution the energy density, pressure and entropy density of ultra-relativistic bosons and fermions are given as

$$\rho_b^{(\text{sm})} = \frac{g\pi^2 T^4}{30}, \quad p_b^{(\text{sm})} = \frac{g\pi^2 T^4}{90}, \quad s_b^{(\text{sm})} = \frac{2g\pi^2 T^3}{45}, \quad (1)$$

and

$$\rho_f^{(\text{sm})} = \frac{7g\pi^2 T^4}{240}, \quad p_f^{(\text{sm})} = \frac{7g\pi^2 T^4}{720}, \quad s_f^{(\text{sm})} = \frac{7g\pi^2 T^3}{180}, \quad (2)$$

where the subscripts b and f stands for bosons and fermions and g stands for any internal degree of freedom of the ultra-relativistic species. It has been shown in [15, 16] that the energy density and the entropy density of the Lee-Wick partners for the bosonic and the fermionic cases are given by [15, 16]:

$$\rho_b^{(\text{LW})} = -g \left(\frac{\pi^2 T^4}{30} - \frac{M^2 T^2}{24} \right), \quad (3)$$

$$p_b^{(\text{LW})} = -g \left(\frac{\pi^2 T^4}{90} - \frac{M^2 T^2}{24} \right), \quad (4)$$

$$s_b^{(\text{LW})} = -g \left(\frac{2\pi^2 T^3}{45} - \frac{M^2 T}{12} \right), \quad (5)$$

and for the fermionic Lee-wick partners one has

$$\rho_f^{(\text{LW})} = -g \left(\frac{7\pi^2 T^4}{240} - \frac{M^2 T^2}{48} \right), \quad (6)$$

$$p_f^{(\text{LW})} = -g \left(\frac{7\pi^2 T^4}{720} - \frac{M^2 T^2}{48} \right), \quad (7)$$

$$s_f^{(\text{LW})} = -g \left(\frac{7\pi^2 T^3}{180} - \frac{M^2 T}{24} \right). \quad (8)$$

In the above equations M is the mass of a generic Lee-Wick partner and as the system is relativistic $T \gg M$. Thus in the toy Lee-Wick model the net energy density, pressure and entropy density of a relativistic bosonic field in early universe composed of the standard

model field and its Lee-Wick partner are [15, 16]:

$$\rho_b = \rho_b^{(\text{sm})} + \rho_b^{(\text{LW})} = \frac{gM^2T^2}{24}, \quad (9)$$

$$p_b = p_b^{(\text{sm})} + p_b^{(\text{LW})} = \frac{gM^2T^2}{24}, \quad (10)$$

$$s_b = s_b^{(\text{sm})} + s_b^{(\text{LW})} = \frac{gM^2T}{12}. \quad (11)$$

In writing the above formulas one assumes the internal degrees of freedom of the standard model bosons and their Lee-Wick partners are the same. For the massless gauge bosons like photon and gluons one has two degrees of freedom whereas their massive Lee-Wick partners seems to have three. Consequently there can be mismatch of degrees of freedom producing a negative energy density. Negative energy density by itself will designate an unstable thermodynamic phase. More over in cosmology, which stems from an underlying theory of general relativity, one must satisfy the weak energy condition which states $\rho \geq 0$. If this condition is not satisfied then the Hubble parameter turns out to be imaginary (for a spatially flat universe) and one loses all kinds of predictive power over the evolving cosmos. The way to avoid the unwanted negative energy condition will be discussed in section 4. In this toy model we will assume that the effect of this extra longitudinal degree of freedom of the Lee-Wick partners of the gauge bosons to be negligible at high temperatures. In the toy model both the massless gauge bosons and their massive partners will have the same internal degrees of freedom. The realistic case where one considers the longitudinal degree of freedom of the massive Lee-Wick partners of the massless gauge bosons will be taken up in section 4.

If there is precisely one Lee-Wick partner for each standard model fermion then the combined energy density, pressure and entropy density of the ultra-relativistic fermionic field and its partner are given as

$$\rho_f = \rho_f^{(\text{sm})} + \rho_f^{(\text{LW})} = \frac{gM^2T^2}{48}, \quad (12)$$

$$p_f = p_f^{(\text{sm})} + p_f^{(\text{LW})} = \frac{gM^2T^2}{48}, \quad (13)$$

$$s_f = s_f^{(\text{sm})} + s_f^{(\text{LW})} = \frac{gM^2T}{24}. \quad (14)$$

On the other hand, if a normal fermionic field has more than one unusual partner, which is the case of a more realistic Lee-Wick standard model [19, 20], then the total energy density can become negative. The realistic case where each standard fermion has two Lee-Wick

partners will be taken up in the penultimate section of this article. From the previous two triplets of equations in the toy Lee-Wick model, one can infer the net energy density and entropy density of a system of ultra-relativistic standard model particles and their partners with mass M_i , as

$$\rho = \frac{T^2}{24} \left[\sum_{i=\text{bosons}} g_i M_i^2 \left(\frac{T_i}{T} \right)^2 + \frac{1}{2} \sum_{i=\text{fermions}} g_i M_i^2 \left(\frac{T_i}{T} \right)^2 \right], \quad (15)$$

$$s = \frac{T}{12} \left[\sum_{i=\text{bosons}} g_i M_i^2 \left(\frac{T_i}{T} \right) + \frac{1}{2} \sum_{i=\text{fermions}} g_i M_i^2 \left(\frac{T_i}{T} \right) \right], \quad (16)$$

where it has been assumed that each standard model boson (or fermion) and its Lee-Wick partner are in a thermal equilibrium but all the different species of particles and their partners may not be sharing the same temperature. Unlike the energy density and entropy density of the standard cosmological model the above results do depend upon the masses of the constituents of the plasma.

If \tilde{M} is the mass of the maximally massive partner field, then we can compactly write for the overall energy density as

$$\rho = \frac{\tilde{M}^2}{24} \tilde{g}_* T^2, \quad (17)$$

where \tilde{g}_* is the effective degree of freedom for energy calculation and the overall entropy density

$$s = \frac{\tilde{M}^2}{12} \tilde{g}_{*s} T, \quad (18)$$

where \tilde{g}_{*s} is the effective degree of freedom for entropy calculation. The effective degrees of freedom are as

$$\tilde{g}_* = \sum_{i=\text{bosons}} g_i \left(\frac{M_i}{\tilde{M}} \right)^2 \left(\frac{T_i}{T} \right)^2 + \frac{1}{2} \sum_{i=\text{fermions}} g_i \left(\frac{M_i}{\tilde{M}} \right)^2 \left(\frac{T_i}{T} \right)^2, \quad (19)$$

and

$$\tilde{g}_{*s} = \sum_{i=\text{bosons}} g_i \left(\frac{M_i}{\tilde{M}} \right)^2 \left(\frac{T_i}{T} \right) + \frac{1}{2} \sum_{i=\text{fermions}} g_i \left(\frac{M_i}{\tilde{M}} \right)^2 \left(\frac{T_i}{T} \right). \quad (20)$$

It is seen from these two expressions that if all the standard particles and their Lee-Wick partners are assumed to be in thermal equilibrium then the values of \tilde{g}_* and \tilde{g}_{*s} are smaller

than the corresponding values of g_* and g_{*s} , where g_* and g_{*s} correspond to the values of the effective degrees of freedom for an equilibrated plasma in conventional cosmology.

There are some interesting properties about the effective degrees of freedom \tilde{g}_* and \tilde{g}_{*s} which are worth noticing. The point which makes the behavior of these effective degrees of freedom different from their standard cosmological counterparts, g_* and g_{*s} , is the dependence of the effective degrees of freedom on the masses of the Lee-Wick partners. In standard cosmological calculations g_* and g_{*s} only depend upon the internal degrees of freedom of the relativistic particles constituting the plasma. In the present case if all the species of the normal particles and their Lee-Wick partners are in thermal equilibrium then it can be seen that $\tilde{g}_* \geq 1$ and $\tilde{g}_{*s} \geq 1$. More over if all the species and their partners are in thermal equilibrium then \tilde{g}_* and \tilde{g}_{*s} do not necessarily decrease as the heaviest Lee-Wick partner becomes non-thermal and drops out of the energy or entropy calculations. This may happen because the heaviest Lee-Wick partner's mass, \tilde{M} , appears in the denominator of the squared mass fractions in the effective degrees of freedom. If the heaviest partner becomes non-thermal then naturally the place of \tilde{M} is replaced by the next heaviest thermalized Lee-Wick partner's mass in the expressions of the effective degrees of freedom. As a result of this the squared mass-fractions appearing in the resulting expressions of \tilde{g}_* and \tilde{g}_{*s} may increase if the mass difference between the two heaviest Lee-Wick partners is high.

3 Cosmological effects of the thermodynamic properties of the Lee-Wick partner infested universe

In the Friedman-Robertson-Walker (FRW) paradigm one can write the line element as

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2, \quad (21)$$

where $a(t)$ is the scale factor of the expanding universe which is spatially flat. The Friedman equation for a spatially flat universe is

$$H^2 = \frac{8\pi}{3M_{\text{Pl}}^2}\rho, \quad (22)$$

where $H = \dot{a}/a$ is the Hubble parameter and the Planck mass M_{Pl} is related to the gravitational constant as $G \equiv 1/M_{\text{Pl}}^2$. The energy density scales with the scale factor as $\rho(t) \propto a(t)^{-3(1+\omega)}$ where the scale factor $a(t) \propto t^{\frac{2}{3(1+\omega)}}$. Here ω is the state parameter which relates the energy density and pressure for a barotropic fluid as $p = \omega\rho$. From the results of the last subsection it can be easily verified that

$$\omega = 1 \quad (23)$$

for a fluid where each ultra-relativistic standard model particle is accompanied by its relativistic Lee-Wick partner. Consequently for a radiation dominated universe infested with Lee-Wick particles one must have

$$\rho(t) \propto a(t)^{-6}, \quad (24)$$

$$a(t) \propto t^{\frac{1}{3}}, \quad (25)$$

a result noted earlier in [15], where in conventional radiation dominated era $\rho(t) \propto a(t)^{-4}$ and $a(t) \propto t^{\frac{1}{2}}$. All of these results point towards a new radiation dominated era of cosmological evolution which differs from the standard radiation dominated ($\omega = 1/3$) and matter dominated ($\omega = 0$) periods.

As in the presence of the thermalized Lee-Wick resonances the entropy density of the universe is given by Eq. (18), thus for an isentropic process the temperature of the universe varies with time as

$$T(t) = \frac{T_0}{a^3(t)}, \quad (26)$$

where T_0 is a constant. In standard cosmology $T \propto a(t)^{-1}$ for an isentropic process. Initially the temperature of the universe is greater than the mass of the heaviest Lee-Wick resonance as a consequence of which the heaviest Lee-Wick resonance remains in thermal equilibrium with the rest of the plasma. In the thermal Lee-Wick resonance dominated radiation era the temperature decreases with time till the temperature of the universe becomes such that $T \sim \tilde{M}$. At this temperature the Lee-Wick resonance with the heaviest mass \tilde{M} decouples from the rest of the plasma. Henceforth the heaviest Lee-Wick partner's mass appearing in the expressions of energy density and entropy density in Eq. (17) and Eq. (18) will get replaced by the next heaviest Lee-Wick partner's mass. When $T \sim \tilde{M}$ and the heaviest Lee-Wick resonance is decoupled, there will be a discontinuity in various thermodynamic parameters of the system which will be discussed later.

From the nature of the scaling of temperature, given in the above equation, it can be inferred that in a universe where the Lee-Wick partners of the standard model particles are also present the time-temperature relationship will change. The exact relationship between cosmic time t and temperature of the universe T is calculated as follows. From Eq. (26) one can immediately write

$$\left(\frac{\dot{T}}{T}\right) = -3H, \quad (27)$$

where H is given by Eq. (22). Using Eq. (22) and Eq. (17) one can write the Hubble

parameter as

$$H = \sqrt{\pi\tilde{g}_*} \frac{T\tilde{M}}{3M_{\text{Pl}}}. \quad (28)$$

Using the above mentioned value of the Hubble parameter one can solve Eq. (27) and get

$$t \sim 10^{-24} (\pi\tilde{g}_*)^{-1/2} \left(\frac{M_{\text{Pl}}}{\tilde{M}} \right) \left(\frac{1\text{GeV}}{T} \right) \text{ sec}. \quad (29)$$

The time-temperature relationship for a Lee-Wick infested radiation dominated universe is distinctly different from the standard time-temperature relationship of the standard radiation dominated universe where

$$t \sim 10^{-6} g_*^{-1/2} \left(\frac{1\text{GeV}}{T} \right)^2 \text{ sec}. \quad (30)$$

The difference between the the two time-temperature relations as given in Eq. (29) and Eq. (30) can have interesting effects in the cosmological evolution of the very early universe. From Eq. (29) one can also write

$$T_{\text{GeV}} \sim 10^{-24} (\pi\tilde{g}_*)^{-1/2} \left(\frac{M_{\text{Pl}}}{\tilde{M}} \right) \frac{1}{t_{\text{sec}}}, \quad (31)$$

where t_{sec} is measured in seconds and T_{GeV} is the temperature of the system in units of GeV.

In standard cosmology the conventional radiation dominated era sets up after reheating which ‘defrosts’ the cold, low-entropy universe at the end of inflationary era. The equilibrated temperature reached after the reheating of the universe, known as the reheat temperature T_{Rh} , is considered as the maximum temperature of the following radiation dominated universe. Considering supersymmetric extension of standard model of particle physics an upper bound on the reheat temperature ($T_{\text{Rh}} < 10^9 \sim 10^{10}$ GeV) comes from the effects of overproduction of gravitinos and their decay products on light element abundances during big bang nucleosynthesis [21]. On the other hand, the Lee-Wick standard model [3], which successfully addresses the ‘hierarchy puzzle’ of the standard model, does not require to include supersymmetric partners. Hence in a cosmological scenario, where the evolution of the universe is described by Lee-Wick extension of standard model, an upper bound on reheat temperature can be set by the constraints arising from (i) generation of Ultra-high energy cosmic rays from super-heavy long-living X –particles during reheating [22], or (ii) imposing naturalness condition on the self-coupling of the inflaton ($\lambda_\phi \sim 10^{-13}$) while estimating the reheat temperature in perturbative reheating scenario (here $T_{\text{Rh}} \sim \sqrt{\Gamma M_{\text{Pl}}}$, Γ being the total

decay rate of inflaton) [23]. Considering these scenarios, preceded by a generic inflationary era, we heuristically set the upper bound on reheat temperature as $T_{\text{Rh}} \lesssim 10^{10}$ GeV. A study of baryogenesis in such a Lee-Wick field infested cosmological scenario is beyond the scope of this paper and requires further investigation. With this upper bound on reheat temperature, one can also determine the age of the universe at the onset of radiation era in standard cosmology using Eq. (30) as

$$t_{\text{rad}} \sim 10^{-27} \text{ sec}, \quad (32)$$

where we have taken $g_* \sim 100$.

In a Lee-Wick partner infested cosmology we refer to an era as a radiation dominated era when all the particles in the cosmic soup are at thermal equilibrium and are all relativistic. The age of the universe at the onset of Lee-Wick infested radiation dominated era, unlike the generic scenario, will depend up on the mass of the heaviest Lee-Wick partner present in the cosmic soup at that time, which can be seen from Eq. (29). Taking $\tilde{M} \sim 10^9$ GeV and $\tilde{g}_* \sim 80$, the reheat temperature ($T_{\text{Rh}} \sim 10^{10}$ GeV) determines the age of the universe at the onset of the Lee-Wick infested radiation era as

$$t_{\text{rad}}^{\text{LW}} \sim 10^{-25} \text{ sec}, \quad (33)$$

whereas a lower mass of the heaviest Lee-Wick partner, say $\tilde{M} \sim 10^5$ GeV, delays the onset of a radiation dominated era where

$$t_{\text{rad}}^{\text{LW}} \sim 10^{-21} \text{ sec}. \quad (34)$$

Now, we will consider $\tilde{M} \sim 10^9$ GeV and $\tilde{g}_* \sim 80$ to demonstrate the evolution of a Lee-Wick infested radiation era since its onset at $t_{\text{rad}}^{\text{LW}} \sim 10^{-25}$ sec. It can be seen from Eq. (31) that as time progresses the temperature of the system decreases and at a certain time, when $T \sim \tilde{M}$, the Lee-Wick partner with the heaviest mass decouples and consequently its contribution will drop out from the expressions of energy density and entropy density of the universe. From then onwards the place of \tilde{M} will be taken up by the mass of the next maximally massive Lee-Wick partner, which we will take as $\tilde{M}' \sim 3 \times 10^8$ GeV. As a consequence of this there will be an abrupt change, specifically a bump in the temperature, when $T \sim \tilde{M}$.

The discontinuity in the temperature of the universe due to decoupling of the heaviest Lee-Wick partner from the cosmic soup is shown in Fig. 1. The magnitude of the bump depends upon the difference of the mass of the maximally massive Lee-Wick partner and the mass of the second maximally massive Lee-Wick partner. The nature of the temperature bump, as the thermal resonances decouple, is complex. As soon as the temperature reaches

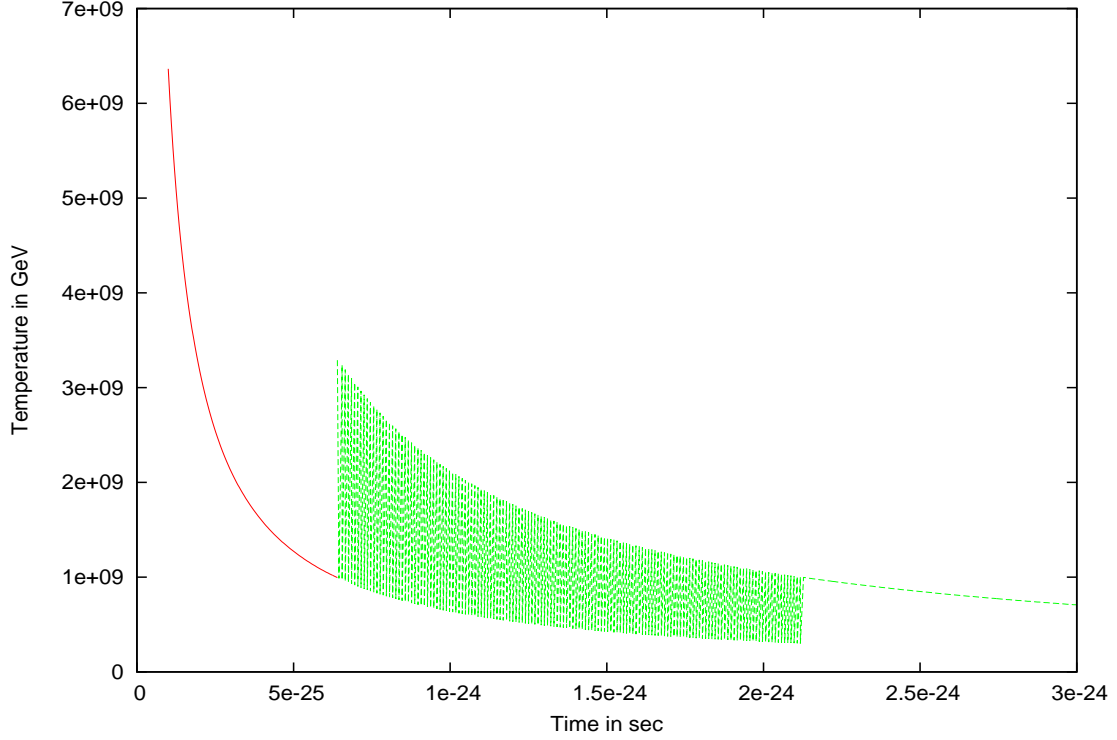


Figure 1: Figure showing the bump in the temperature as the heaviest Lee-Wick partner with mass 10^9 GeV becomes non-thermal and its position is taken up by the next heaviest Lee-Wick partner of mass 3×10^8 GeV. For a simple illustration the effective degrees of freedom are assumed to remain constant in the process, $\tilde{g}_* \sim 80$. The green region in the plot represents a region of rapid and violent temperature fluctuations where the system has gone out of equilibrium.

\tilde{M} there is an abrupt rise in the temperature which, if greater than \tilde{M} , re-thermalizes the heaviest Lee-Wick mode once again. As the temperature of the re-thermalized universe tries to come down and the temperature reaches \tilde{M} again there is a second bump which again re-thermalize the heaviest Lee-Wick partner. Consequently there are rapid and violent temperature fluctuations which takes the system out of thermal equilibrium. The fact that the system goes out of equilibrium can also be verified by the entropy bump in Fig. 2 where it is shown that there is an abrupt bump in the entropy of the universe as the heaviest Lee-Wick resonance decouples. Equilibrium is regained once the resultant temperature is smaller than \tilde{M} such that the maximally massive Lee-Wick partner is not re-thermalized. This effect is portrayed by the shaded region in Fig. 1. The entropy bump in Fig. 2 can also

be explained by an analogous analysis.

It is interesting to note here that as the heaviest Lee-Wick partner decouples from the cosmic plasma and ceases to be thermalized, it will leave behind its standard model partner in the cosmic plasma which being very light will still be thermalized and will contribute to the energy and entropy density of the radiation dominated universe. Thus the decoupling of the heaviest of the Lee-Wick partners will change the expressions of the energy density and entropy density from the forms they had in Eq. (17) and Eq. (18). After some of the Lee-Wick partners become non-thermal, the form of the energy density and the entropy density will look like

$$\rho' = \frac{\tilde{M}'^2}{24} \tilde{g}'_* T'^2 + \frac{\pi^2 T'^4}{30} g'_* , \quad (35)$$

$$s' = \frac{\tilde{M}'^2}{12} \tilde{g}'_{*s} T' + \frac{2\pi^2 T'^3}{45} g'_{*s} . \quad (36)$$

These expressions show that after the decoupling of few heaviest Lee-Wick partners the universe will be in a mixed state where the two components of the mixture have different forms of thermodynamic parameters. The first terms on the right hand sides of the equations comes from that part of the plasma which consists of those standard model particles and their Lee-Wick partners which are still thermalized at that temperature T' . The second term of the right hand sides of the equations comes from a plasma constituted by only standard model particles whose Lee-Wick partners have already decoupled from the cosmic plasma. As the Lee-Wick partners are heavier than their standard model partners it is safe to assume that these standard model particles are still relativistic at temperature T' . In the above equations we assume that \tilde{M}' is the mass of the maximally massive Lee-Wick partner at the corresponding temperature T' . The effective degrees of freedom changes to \tilde{g}'_* and \tilde{g}'_{*s} from their previous values. Also, g'_* and g'_{*s} account for the effective degrees of freedom of those standard model particles whose Lee-Wick partners have decoupled.

If very few of the Lee-Wick partners have become non-thermal then $\tilde{g}'_* \gg g'_*$ and $\tilde{g}'_{*s} \gg g'_{*s}$ and more over if the temperature T' happens to be marginally greater than \tilde{M}' , $T' \gtrsim \tilde{M}'$, then the presence of the Lee-Wick partners predominantly shape the thermodynamics of the plasma, which can be seen from Eq. (35), and consequently the evolution of the thermodynamic parameters follows the laws as discussed in this section. In such a case the equation of state of the plasma will change slightly and $\omega \lesssim 1$. A simple situation of this kind is portrayed in the temperature and entropy bumps in Fig. 1 and Fig. 2. In the figures \tilde{g}_* and \tilde{g}_{*s} remain approximately constant after the heaviest Lee-Wick partner become non-thermal and the standard model particle which loses its partner is a boson with $g = 2$. As more and more Lee-Wick partners become non-thermal we will have $\tilde{g}'_* \gtrsim g'_*$ and $\tilde{g}'_{*s} \gtrsim g'_{*s}$, but once

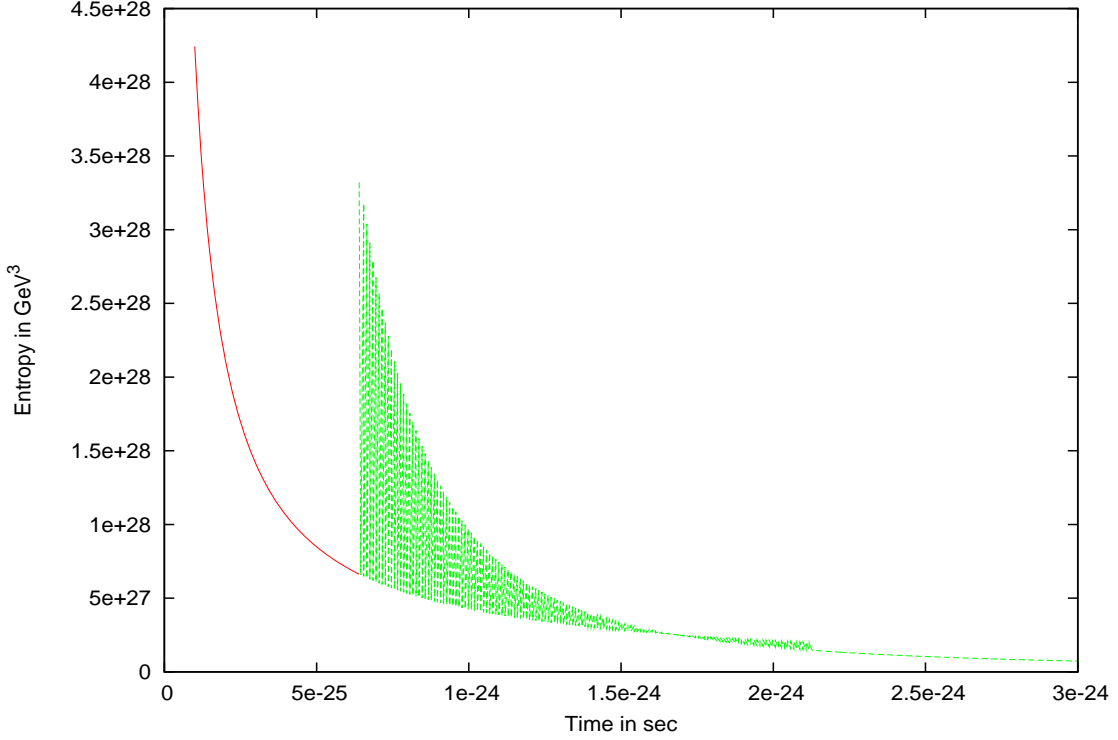


Figure 2: Figure showing the bump in the entropy density of the universe as the heaviest Lee-Wick partner with mass 10^9 GeV becomes non-thermal and its position is taken up by the next heaviest Lee-Wick partner of mass 3×10^8 GeV. The effective degrees of freedom are assumed to remain constant in the process as $\tilde{g}_{*s} \sim 80$. The standard model particle freed of its partner is assumed to be a boson with $g = 2$. The spiked green region in the plot represents a region of rapid and violent entropy fluctuations where the system has gone out of equilibrium.

$T' \gtrsim \tilde{M}'$ the second terms of Eq. (35) and Eq. (36) start to dominate and consequently the thermodynamics of the universe transforms from the unusual Lee-Wick particle dominated phase to the standard radiation dominated phase where now $\omega \sim 1/3$. The point to be noted is that the thermodynamic development of the universe can become the standard one after some of the Lee-Wick partners become non-thermal and the rest are still there in the plasma. To have the universe evolve as the standard radiation dominated one with $\omega \sim \frac{1}{3}$ one should have the standard model particles, which are devoid of their Lee-Wick partners,

to dominate the energy density of the universe. From Eq. (35) this condition yields

$$T' > 0.36 \left(\frac{\tilde{g}'_*}{g'_*} \right)^{\frac{1}{2}} \tilde{M}'. \quad (37)$$

We require the generic radiation dominated era to set well before neutrino decoupling so that the general scenario of the radiation dominated universe is not much affected by the presence of the Lee-Wick partners. Hence we set $t \sim 1$ sec as the time of the onset of the generic radiation dominated universe with $\omega \sim \frac{1}{3}$. Before the onset of the generic radiation domination the evolution of the universe is governed by the presence of the heavy Lee-Wick resonances and thus the time-temperature relation will follow the form given in Eq. (31). An upper bound on mass of the heaviest Lee-Wick partner, which can still remain in the thermal cosmic soup even after the onset of a generic radiation dominated universe at $t \sim 1$ sec, thus can be determined by Eq. (31) and Eq. (37) as

$$\tilde{M}' < 1.4 \times 10^{-2} \text{ GeV}, \quad (38)$$

where we have assumed for simplicity $\tilde{g}'_* \approx g'_* \approx 60$ at the time of onset of generic radiation domination.

4 A more realistic approach

There are some subtle issues related to the number of fermionic Lee-Wick partners accompanying a normal fermion as discussed. The subtlety of the issue is related to how many Lee-Wick partners does a chiral fermion have? Initially it was supposed that there are two but those two cannot be treated as independent degrees of freedom as they are related to each other in the higher derivative form of the theory. The main reason for declining the independence of the two Lee-Wick fermion partners is related to the emergence of negative energy density. The issue remains subtle as can be seen in the discussions in [19, 20]. In the present section we will not go into a theoretical debate about this issue, but we will present a picture of the possible thermodynamics of the universe if both the fermionic partners of a standard chiral fermion were really independent degrees of freedom in the auxiliary field approach. It can be shown that although this scenario is precarious, as it can potentially lead to an early universe with negative energy density, there can be some conditions during cosmological reheating which can evade the difficulties and produce a workable model of early universe thermodynamics.

More over in the toy model of the Lee-Wick cosmology as discussed in the sections before we ignored the longitudinal mode of polarization of the massive Lee-Wick partners of

massless vector bosons. In this section we will like to incorporate their effect in the energy density and other relevant thermodynamic parameters. If one assumes that both the Lee-Wick partners of each Standard model fermion are of equal mass and considerably heavier than their standard model partners, then considering the bosons and fermions with their Lee-Wick partners altogether, one can write

$$\rho = \frac{\tilde{M}^2}{24} \tilde{g}_{*N} T^2 - \frac{7\pi^2}{240} \tilde{g}_F T^4 - \frac{\pi^2}{30} n T^4, \quad (39)$$

where the new degrees of freedom \tilde{g}_{*N} and \tilde{g}_F are given as

$$\tilde{g}_{*N} = \sum_{i=\text{bosons}} g_{iN} \left(\frac{M_i}{\tilde{M}} \right)^2 \left(\frac{T_i}{T} \right)^2 + \sum_{i=\text{fermions}} g_i \left(\frac{M_i}{\tilde{M}} \right)^2 \left(\frac{T_i}{T} \right)^2, \quad (40)$$

where g_{iN} for bosonic particles stands for the internal degrees of freedom g_i for the partners of massive standard bosons (may be 2 or 1), while for standard massless vector boson partners it equals $g_i + 1$ where primarily $g_i = 2$. The unpaired fermionic contribution comes with \tilde{g}_F where

$$\tilde{g}_F = \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T} \right)^4, \quad (41)$$

where \tilde{g}_F solely arises from the unpaired fermionic Lee-Wick partners of the standard model particles. The quantity n is defined as

$$n = \sum_{i=\text{massive vect. bosons}} \left(\frac{T_i}{T} \right)^4. \quad (42)$$

Here T_i is the temperature at which the i^{th} massive Lee-Wick vector boson partner of a standard massless gauge boson is equilibrated and n denotes the number of massive vector boson partners of massless standard gauge bosons if all the species are in thermal equilibrium at the same temperature T . The sum appearing in Eq. (42) does not include all the massive Lee-Wick vector boson partners, it includes only those which are partners of massless standard gauge bosons.

Assuming all the particle species and their partners are in thermal equilibrium it can be verified from Eq. (39) that if the temperature of the universe satisfies the following inequality

$$T \leq \sqrt{\frac{5 \tilde{g}_{*N}}{4\pi^2(n + \frac{7}{8} \tilde{g}_F)}} \tilde{M}, \quad (43)$$

then the energy density of the universe can be positive. The inequality above is very restrictive as for thermal Lee-Wick partners $T \geq \tilde{M}$. Combining these two inequalities one can say that the very early universe can have a very small positive energy density only if

$$\tilde{g}_{*N} \geq \frac{4\pi^2}{5} \left(n + \frac{7}{8} \tilde{g}_F \right), \quad (44)$$

which sets a constraint on the internal degrees of freedom available for the particles inhabiting the universe during these early times. Eq. (44) is a difficult one to satisfy as \tilde{g}_{*N} must be a small number if the Lee-Wick partner masses have a stiff hierarchy as \tilde{g}_{*N} contains the square of the ratios of $(M_i/\tilde{M})^2$, where as \tilde{g}_F and n does not have any such factors. Before we comment on the possibility of any cosmological phase where one has two Lee-Wick partners of chiral fermions and considers the longitudinal degrees of freedom of massive Lee-Wick gauge fields it is better to have an expression of entropy of such a system. The pressure of such a plasma which consists of two Lee-Wick partners of one standard chiral fermion is given by

$$p = \frac{\tilde{M}^2}{24} \tilde{g}_{*N} T^2 - \frac{7\pi^2}{720} \tilde{g}_F T^4 - \frac{\pi^2}{90} n T^4. \quad (45)$$

It must be noted that if the energy density of the Lee-Wick partner infested universe is positive then the pressure of the same universe must be positive. From the expressions of the energy density and pressure one can calculate the entropy density of such a plasma as

$$s = \frac{\tilde{M}^2}{12} \tilde{g}_{*N} T - \frac{7\pi^2}{180} \tilde{g}_F T^3 - \frac{2\pi^2}{45} n T^3. \quad (46)$$

From the above expression one can easily see that the entropy density is not positive definite at high temperatures and if it is indeed positive it can be very small. If one has to think of a radiation dominated phase at the time of reheating after inflation one must have to generate entropy. The Lee-Wick partners of the fermions and massless gauge bosons does not allow this high entropy generation during reheating.

In the present scenario one sees that if there are two Lee-Wick fermionic partners for each standard fermion both energy density and entropy density remains very low. There can be some interesting cases where the above difficulties can be circumvented. In the following discussion we briefly mention some of these conditions :

1. If the early universe had considerably more number of heavy bosons as compared to the total number of chiral fermions and massless gauge bosons then the condition set by Eq. (44) can be satisfied. Such a scenario can be related to the preheating scenario

[24] where the coherently oscillating classical inflaton field decays very rapidly into many heavy bosons due to broad parametric resonances. The bosons outnumbered the fermions during preheating as the Pauli exclusion principle prohibits explosive creation of fermions. As this process of creating particles via preheating is very rapid, the heavy bosons thus produced are initially far away from thermal equilibrium. But at a later stage when the particles initially thermalize during the generic reheating scenario then it may happen that few (or none) of the fermion species (and their partners) are initially thermalized and one obtains an over abundance of bosons over fermions. The other (or all) fermions and massless vector bosons (and possibly their partners) thermalize later when the Lee-Wick phase has evolved into the standard radiation dominated phase. In this case the energy density and the entropy density will predominantly have the heavy bosonic parts and their forms will be similar to the energy density and entropy density as given in Eq. (17) and Eq. (18) where now \tilde{g}_* and \tilde{g}_{*s} have to be replaced by their corresponding values which have predominantly heavy bosonic degrees of freedom. The thermodynamic evolution of such an universe matches with the one described in the beginning of the last section.

2. There is another possibility by which the constraint in Eq. (44) can be satisfied and there can be high entropy generation at the time of reheating. If the Lee-Wick partners of the chiral fermions and the massless gauge bosons are much heavier and the lightest fermionic Lee-Wick partner's mass and the lightest Lee-wick massive vector boson (which is a partner of a standard gauge boson) mass are greater than the reheat temperature T_{Rh} then at reheating temperature the Lee-Wick partners of the fermions and gauge bosons must have decoupled from the plasma and consequently one must have only the heavy bosons with their Lee-Wick partners and standard fermions and gauge bosons contributing to the energy density of the universe. This scenario is favorable if one has a low reheat temperature $\sim 10^{6-5}$ GeV or less. In this case the energy density and entropy density will have no contributions from the Lee-Wick partners of the, fermions and the massless gauge bosons, and the energy density and entropy density will be

$$\rho = \frac{\tilde{M}^2}{24} \tilde{g}_{*N}^b T^2 + \frac{\pi^2}{30} \left(g_0^b + \frac{7}{8} \tilde{g}_F \right) T^4, \quad (47)$$

which is positive definite. The entropy will also turn out to be positive definite. The factor \tilde{g}_{*N}^b is exactly the same as \tilde{g}_{*N} except that it does not have the fermionic and gauge boson part. The gauge bosons internal degrees of freedom are encapsulated in the term g_0^b . The analysis of the thermodynamic evolution of such an universe is qualitatively similar to the analysis given in the previous section, specifically after

Eq. (35) and Eq. (36). As presently there is no theoretical model to predict the range of masses of the Lee-Wick partner so the assumption of having heavier fermionic and gauge boson partners remains a possibility which can be tackled by future experiments.

Presently, the bounds on the electroweak Lee-Wick sector [25, 26, 27] predict Lee-Wick partners masses around 3 TeV or above. There is an interesting observation regarding the production mechanism of Lee-Wick fermion resonances [27, 28]. The Lee-Wick fermion resonances when produced will generally occur in pairs. Single production of fermionic Lee-Wick resonances are suppressed by the Lee-Wick fermions' mass. In a realistic situation this implies that most of the heavy Lee-Wick fermion partners will not be in thermal equilibrium if the reheat temperature is low. There will be more single production of Lee-Wick bosonic resonances compared to fermionic ones if the bosonic modes have masses comparable or lower than the fermionic partner masses.

The present model of Lee-Wick cosmology offers an early radiation dominated universe whose energy density and entropy density can be positive but they can be very small. If any of the above conditions are fulfilled then one can circumvent this problem. The conditions stated above are quite restrictive and consequently a Lee-Wick phase in the early universe can definitely lead to a different kind of cosmology as compared to the standard one.

5 Discussion and conclusion

In standard radiation domination a usual particle can annihilate itself by interacting with its antiparticle and producing photons in thermal equilibrium. The entropy density remains constant and consequently the temperature scales such that $Ta(t)$ remains constant during this process of annihilation. In the Lee-Wick universe these conventional scenarios do not hold any more. The simple reason why they do not hold is related to the fact that the thermalized Lee-Wick resonances have negative entropy density and consequently when these thermal modes decouple they produce entropy. Entropy production designates an out of equilibrium situation where the system is heated up. As a result of this influx of energy the temperature of the system also shoots up. The decoupling of any relativistic Lee-Wick thermal resonance is an out of equilibrium process which will reheat the universe. This reheating of the universe results in an increased temperature of the universe.

From the nature of the Lee-Wick theories it was predicted that these theories violate causality, as one has to apply future boundary conditions to eradicate run away solutions [2]. Lee and Wick claimed and showed that violation of causality will not be observable in low energy experiments. Application of the Lee-Wick paradigm in the very early universe cosmology opens up a new question. It is a priory not known how the accusal behavior of

the Lee-Wick sector is going to affect the causal history of the universe. The present authors do not have an answer to this issue and it remains as a challenge to be addressed in the future.

In the present article we initially present a toy model of Lee-Wick thermodynamics where each standard boson or fermion has one Lee-Wick partner respectively and more over the partners of the standard gauge bosons have only two degrees of freedom. The qualitative features coming out of this model predict the general nature of a Lee-Wick resonance dominated early universe. In the realistic model one considers two Lee-Wick fermion partners for each standard fermion [19, 20] and also takes care of the longitudinal degree of freedom of the massive Lee-Wick partners of the standard gauge bosons. In the realistic Lee-Wick model it was shown that unless one uses some specific assumptions about the bosonic and fermionic degrees of freedom of the early universe it becomes difficult to produce enough amount of energy density and entropy density during reheating. One requires, in such a case, the bosonic degrees of freedom (arising from the massive bosons) to outnumber the fermionic degrees of freedom to achieve a workable model of the early universe. The general nature of evolution in Lee-Wick cosmology remains the same as was presented in the toy model under various limiting conditions. Each time a Lee-Wick partner decouples from the cosmic plasma there will be a mini reheating. The central idea of the present work is that if the Lee-Wick partners are thermalized at reheating epoch after inflation then there will be a series of explosive events when the temperature and entropy of the universe will increase and consequently the system will go out of thermal equilibrium. These events will happen at times when Lee-Wick partners decouple from the cosmic plasma. The realistic model of Lee-Wick theories which require two fermionic partners and three degrees of freedom of the Lee-Wick partners of gauge bosons demand a mass hierarchy of the partners where preferably the fermionic partners are heavier or comparable in mass to their bosonic counterparts if the reheat temperature is quite low ($T_{\text{Rh}} \sim 10^5 \text{ GeV}$). In this case one has to assume that the mass of the gauge boson partners to be quite high when compared to the reheating temperature. On the other hand, if the reheat temperature is high ($T_{\text{Rh}} \sim 10^{10} \text{ GeV}$), then a preheating scenario [24], which overproduces massive bosons than fermions and is followed by reheating and thermalization of particles, is preferable to describe an early universe dominated by Lee-Wick resonances.

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References

- [1] T. D. Lee and G. C. Wick, Nucl. Phys. B **9**, 209 (1969).
- [2] T. D. Lee and G. C. Wick, Phys. Rev. D **2**, 1033 (1970).
- [3] B. Grinstein, D. O’Connell and M. B. Wise, Phys. Rev. D **77**, 025012 (2008) [arXiv:0704.1845 [hep-ph]].
- [4] C. D. Carone and R. F. Lebed, Phys. Lett. B **668**, 221 (2008) [arXiv:0806.4555 [hep-ph]].
- [5] C. D. Carone and R. F. Lebed, JHEP **0901**, 043 (2009) [arXiv:0811.4150 [hep-ph]].
- [6] C. D. Carone, Phys. Lett. B **677**, 306 (2009) [arXiv:0904.2359 [hep-ph]].
- [7] C. D. Carone and R. Primulando, Phys. Rev. D **80**, 055020 (2009) [arXiv:0908.0342 [hep-ph]].
- [8] E. Alvarez, E. C. Leskow and J. Zurita, Phys. Rev. D **83**, 115024 (2011) [arXiv:1104.3496 [hep-ph]].
- [9] F. Krauss, T. E. J. Underwood and R. Zwicky, Phys. Rev. D **77**, 015012 (2008) [Erratum-ibid. D **83**, 019902 (2011)] [arXiv:0709.4054 [hep-ph]].
- [10] T. Figy and R. Zwicky, JHEP **1110**, 145 (2011) [arXiv:1108.3765 [hep-ph]].
- [11] Y. F. Cai, T. t. Qiu, R. Brandenberger and X. m. Zhang, Phys. Rev. D **80**, 023511 (2009) [arXiv:0810.4677 [hep-th]].
- [12] J. Karouby and R. Brandenberger, Phys. Rev. D **82**, 063532 (2010) [arXiv:1004.4947 [hep-th]].
- [13] J. Karouby, T. Qiu and R. Brandenberger, Phys. Rev. D **84**, 043505 (2011) [arXiv:1104.3193 [hep-th]].
- [14] I. Cho and O. K. Kwon, JCAP **1111**, 043 (2011) [arXiv:1109.5753 [gr-qc]].
- [15] B. Fornal, B. Grinstein and M. B. Wise, Phys. Lett. B **674**, 330 (2009) [arXiv:0902.1585 [hep-th]].
- [16] K. Bhattacharya and S. Das, Phys. Rev. D **84**, 045023 (2011) [arXiv:1108.0483 [hep-ph]].

- [17] R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. **187**, 345 (1969).
- [18] D. G. Boulware and D. J. Gross, Nucl. Phys. B **233**, 1 (1984).
- [19] M. B. Wise, Int. J. Mod. Phys. A **25**, 587 (2010) [arXiv:0908.3872 [hep-ph]].
- [20] J. R. Espinosa, B. Grinstein, Phys. Rev. **D83**, 075019 (2011). [arXiv:1101.5538 [hep-ph]].
- [21] J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B **145**, 181 (1984).
- [22] V. A. Kuzmin and V. A. Rubakov, Phys. Atom. Nucl. **61**, 1028 (1998) [Yad. Fiz. **61**, 1122 (1998)] [astro-ph/9709187].
- [23] B. R. Greene, T. Prokopec and T. G. Roos, Phys. Rev. D **56**, 6484 (1997) [hep-ph/9705357].
- [24] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994) [hep-th/9405187].
- [25] E. Alvarez, C. Schat, L. Da Rold and A. Szynkman, arXiv:0810.3463 [hep-ph].
- [26] T. E. J. Underwood and R. Zwicky, Phys. Rev. D **79**, 035016 (2009) [arXiv:0805.3296 [hep-ph]].
- [27] E. Alvarez, L. Da Rold, C. Schat and A. Szynkman, JHEP **0804**, 026 (2008) [arXiv:0802.1061 [hep-ph]].
- [28] T. G. Rizzo, JHEP **0706**, 070 (2007) [arXiv:0704.3458 [hep-ph]].